BIOINSPIRATION - SHAPE OPTIMIZATION in BIOLOGICAL STRUCTURES REVISITED

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Preamble

This article is based on an invited lecture that I gave at the NAFEMS World Congress at Newport, Rhode Island in 1999 [1]. The topic is a "hobby" that I have pursued throughout my career. For this article, I have expanded and enhanced the original content. In particular, I have taken the opportunity to add colour, which greatly enhances the illustrations. Various exemplars of shape optimization and weight efficiency in biological structures are examined. The inspiring structures and shapes briefly examined include the honeycomb of the bee, the rachis of a bird feather; spider's silk and webs, human heart valves, trees and the Baud curve. The appropriateness of the terms "optimization" and "fitness for purpose" are also briefly examined and the difficulty of establishing objective functions and constraints for possible optimization scenarios is discussed. This then is a brief summary of a diverse range of work, with a focus on structures and materials.

1. A Question to Ponder … *Are Biological Structures Optimum?*

Michael French [2], stated that *living organisms are examples of design strictly for function, the product of blind evolutionary forces rather than conscious thought, yet far excelling the products of engineering.* How therefore, can the structure of such organisms possibly represent optimum solutions, given the lack of conscious thought in their development ? One answer is of course provided by Darwin's [3] theory of natural selection. The cumulative effects of hereditary variation, coupled with a natural selection process, it is argued, inevitably leads to organisms and structures that are fitter for the purpose of survival. The process of cumulative selection is effectively explained by Dawkins in his popular classic The Blind Watchmaker [4]. Whether, in the scheme of things, biological structures are considered simply *adequate* or *optimum* is open to debate and depends to some extent on how the problem is posed. The evolution of a species by natural selection is inherently linked to the competition that it experiences. The extent of evolution (and hence development towards an optimum) is therefore inextricably linked to such pressures. It must also be borne in mind, that biological structures evolve gradually... the option of a revolutionary change in approach is generally not available. That being the case, then it may be argued that at most, natural structures probably represent *local optima* or *best* solutions. Optimization scenarios can certainly be postulated, but whether the goals and constraints proposed are the criteria that influence the development of the organism may be difficult to ascertain.

Although examples in nature are common where the *aesthetic* qualities of a particular organism clearly affects success (mainly with respect to attractiveness to members of the same species for mating purposes or to a different organism completely as in the case of pollination), it is probably true to say that the form of a great many natural structures arrives, more often than not, as a consequence of economy rather than aesthetics. As Newton noted in his work Philosophiae Naturalis Principia Mathematica [5] ... *the phiosophers say that Nature does nothing in vain, and more is vain when less will serve*; *for*

Nature is pleased with simplicity, and affects not the pomp of superfluous causes. There are clearly energy costs associated with growing bigger, stronger and with moving about. In the latter case particularly, it is obvious that mass will play an important role in the success of such organisms. It is therefore perhaps not surprising that a diverse array of fascinating examples of weight efficient structures and materials exists in nature.

In many instances it is not at all obvious that an optimum or even a best solution has been achieved. However, the fact that any such structure or organism does not appear to be optimum, is often due to our lack of understanding of the particular optimization objective function(s) and constraints. This same observation was originally made by D'Arcy Thompson in his treatise On Growth and Form [6] *We have dealt with problems of maxima and minima in many simple configurations, where form alone seemed to be in question; and when we meet with the same principle again wherever work has to be done and mechanism is at hand to do it. That this mechanism is the best possible under all the circumstances of the case, that its work is done with a maximum of efficiency and at a minimum of cost, may not always lie within our range of quantitative demonstration, but to believe it to be so is part of our common faith in the perfection of Nature's handiwork.*

Optimization of shape and form in nature embraces far more than the simple goal of minimum mass so often encountered in engineering structures. However, as has already been stated, for many biological structures it will be metabolically advantageous to reduce weight and many of the examples contained herein are presented in this context. The natural constraints arising from material and structural performance may also be different to those normally encountered by the engineer. The requirements for reproduction and growth impose significant constraints on the variables available for a natural solution. It may be argued that the ultimate unconscious goal for all biological structures is to **survive and reproduce** ... not an option available in the current generation of engineering optimization software! In biology, the concepts of survival and reproduction are referred to as *fitness* and Alexander [7] discussed the difficulty of using this as the objective function in a range of optimization studies in animals.

Although many biological structures have the ability to repair damage, it is also apparent that catastrophic failures do occur. The fact that trees blow down in the wind and animals break their bones, does not necessarily mean that such structures are not fit for purpose or even optimum. The problem often lies in our understanding of the objective function and constraints. Obviously biological structures normally have reserves of strength against the various environmental loadings and failure mechanisms that they may be subjected to. However, such reserves of strength are not predetermined by some design standard, as is generally the case in engineering, but are determined by natural selection for each individual case [8][9]. It is argued that nature in effect *chooses* not to design against such eventualities. Being *stronger and stiffer* will incur an energy or metabolic cost and if such cost impairs an organism's performance in other areas e.g. to gain food and to reproduce, then this may affect its overall chances of survival in competition with other species ... the solution is often seen to be a compromise and may not be obvious. Thus in nature's complex survival algorithm, the frequency of structural failure may be tolerated to a higher degree than that acceptable to the engineer. High incidences of failure in a particular biological structure may also be evidence of a change in environment and a species' inability to adapt quickly enough.

In its simplest form, a shape optimization problem will have the following characteristics :

A goal or objective ... e.g. minimum mass
Geometry & material variables ... with limits on range
Constraints ... e.g. on stress, strain, deflection etc
Loads & environmental effects ... e.g. self weight, pressure, temperature etc

Optimisation scenarios involving biological structures may often bring the additional complication of constraints and variables that may vary with time. In addition, the natural forces involved may vary in a random and non-linear manner.

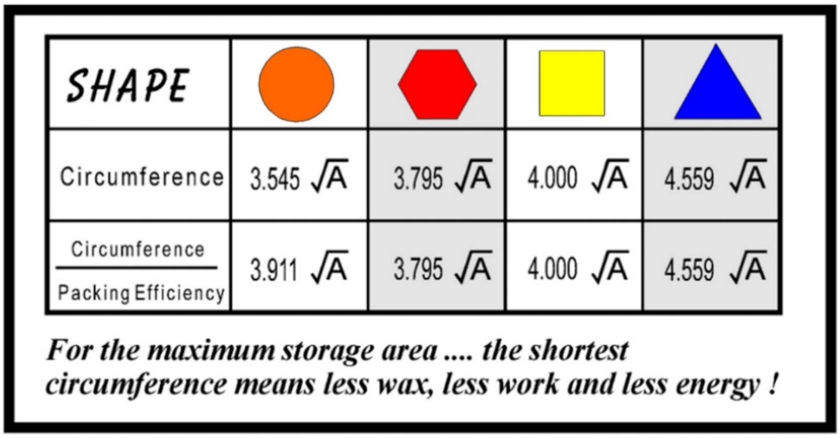
Interestingly, optimization procedures based on biological growth have been developed by Umetami & Hirai [22], Mattheck [23][24], amongst others.

While it may be a challenge to demonstrate that the solution achieved is a true optimum, it is often quite straightforward for the user to demonstrate quantifiably that the solution is a better one (for the scenario posed) and many engineers may be content at that.

2. A Historical Exemplar … *The Honeycomb of the Bee.*

The comb of the honeybee has fascinated man for centuries and D'Arcy Thompson [6] cites early references by Virgil, Ausonius (c AD370), Pliny and Pappus of Alexandria and devoted some 19 pages of his seminal work On Growth and Form to this particular topic. Charles Darwin [3] also studied the subject extensively and 9 pages of The Origin of Species provides a record of his observations. Darwin notes that ... *he must be a dull man who can examine the exquisite structure of a comb, so beautifully adapted to its end, without enthusiastic admiration.* Ausonius a latin poet, wrote of *Geometrica Forma Favorum* and Papus, a mathematician, wrote that the *bees were endowed with a certain geometrical forethought.*

The comb of the honeybee has probably received greater interest from scientists and mathematicians over the centuries than any other natural structure. The references are too numerous to mention and without doubt stems from man's historical associations with the bee as a source of honey. Of particular historical note is D'Arcy's reference to the studies of Pappus of Alexandria around AD3 .... *his conclusions regarding the hexagonal shape of the cell arising from a consideration of economy of wax, D'Arcy notes, is* ***probably the earliest record of such a minimisation principle*** *and predates the principle of least action that guided Leibniz, Maupertuis and other 18th century physicists, mathematicians and philosophers such as Bernoulli, Euler, Lagrange and Koenig*.



SHAPE	Circle	Hexagon	Square	Triangle
Circumference	$3.545 \sqrt{A}$	$3.795 \sqrt{A}$	$4.000 \sqrt{A}$	$4.559 \sqrt{A}$
Circumference / Packing Efficiency	$3.911 \sqrt{A}$	$3.795 \sqrt{A}$	$4.000 \sqrt{A}$	$4.559 \sqrt{A}$

For the maximum storage area .... the shortest circumference means less wax, less work and less energy !

Figure 1 : The optimum cell shape for a honeycomb

That the hexagonal shaped cell is the optimum in terms of honey storage for the least quantity of wax used, is simply illustrated in Figure 1. For a given cross-sectional area *A*, the circumference of the hexagon is the smallest, after due allowance has been made for the packing efficiency of the circle.

The cell of the comb of the honeybee was also the basis of a celebrated optimization problem for 18th century mathematicians. The problem in this case concerned the shape of the bottom of the cell. The problem was effectively stated by the French naturalist Rene Antoine Ferchault R'eaumur and became known as **The Problem of the Bees** … *A cell of regular hexagonal cross-section is closed by three equal and equally inclined rhombs: calculate the smaller angle of the rhombs when the total surface area of the cell is the least possible*. The first widely published value had previously been attributed to an astronomer working in Paris in the 1730's named Maraldi and the angle of 70 degrees 32 minutes became known as the Maraldi Angle. Maraldi also noted that this was *exactly* the angle built by the bees. Subsequent work has rightly noted that the use of the term *exact* was not appropriate, given the variability of the natural structure. Furthermore, the precise angle was later shown to be that of a rhombic dodecahedron and given by cos-1(1/3). Koenig, the Swiss mathematician, solved the problem using calculus in 1739 and then asserted that the bees had solved a problem beyond the reach of the old geometry and requiring the methods of Newton and Leibniz. However Colin MacLaurin, the Professor of Mathematics at Edinburgh University, demonstrated [10] in 1743 that a geometry based solution was possible and concluded his presentation to the Royal Society with the observation that *what is most beautiful and regular, is also found to be most useful and excellent*.

These observations led to equally fascinating studies on insect intelligence and Bernard le Bovyer de Fontenelle, the French philosopher, is credited with the judgement in which the bees were denied intelligence but were nevertheless found to be blindly using the highest mathematics by divine guidance and command ! The final compliment paid to the comb of the honeybee, is made by Charles Darwin when he wrote [3] ... *beyond this stage of perfection in architecture, natural selection could not lead; for the comb of the hive-bee, as far as we can see, is absolutely perfect in economizing labour and wax*.

D'Arcy Thompson stated in the introduction to his 1942 edition of On Growth and Form

[6] that … *It is no wonder if new methods, new laws, new words, new modes of thought are needed when we make bold to contemplate a universe within which all Newton's is but a speck*. It is with some pride therefore that a solution to the problem of the bees, using a geometry modeler-based shape optimization tool, is presented in Figure 2 … a new method using new words and new modes of thought. A shape optimization benchmark with a finer pedigree surely could not be found! Furthermore, it is demonstrated in Figure 3, using FEM, that not only is this a minimum mass solution, it is also a minimum stress solution for a cell subjected to hydrostatic pressure loading of honey!

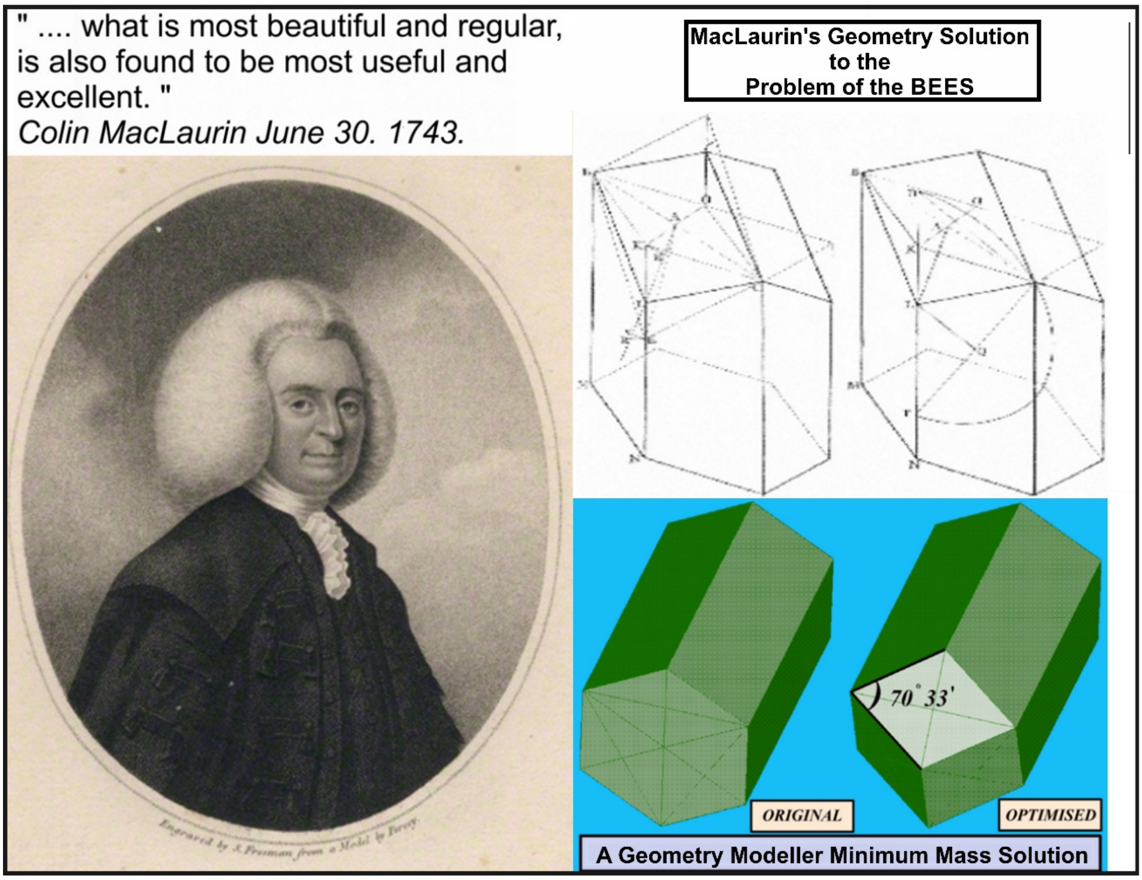


Figure 2 : Geometry modeler minimum mass solution for honeybee cell

The angles shown in Figures 2 & 3 were obtained using an adaptive-p finite element system *(Mechanica*) with a goal of minimum mass and with the end points set as translation variables along the axis of the cell.

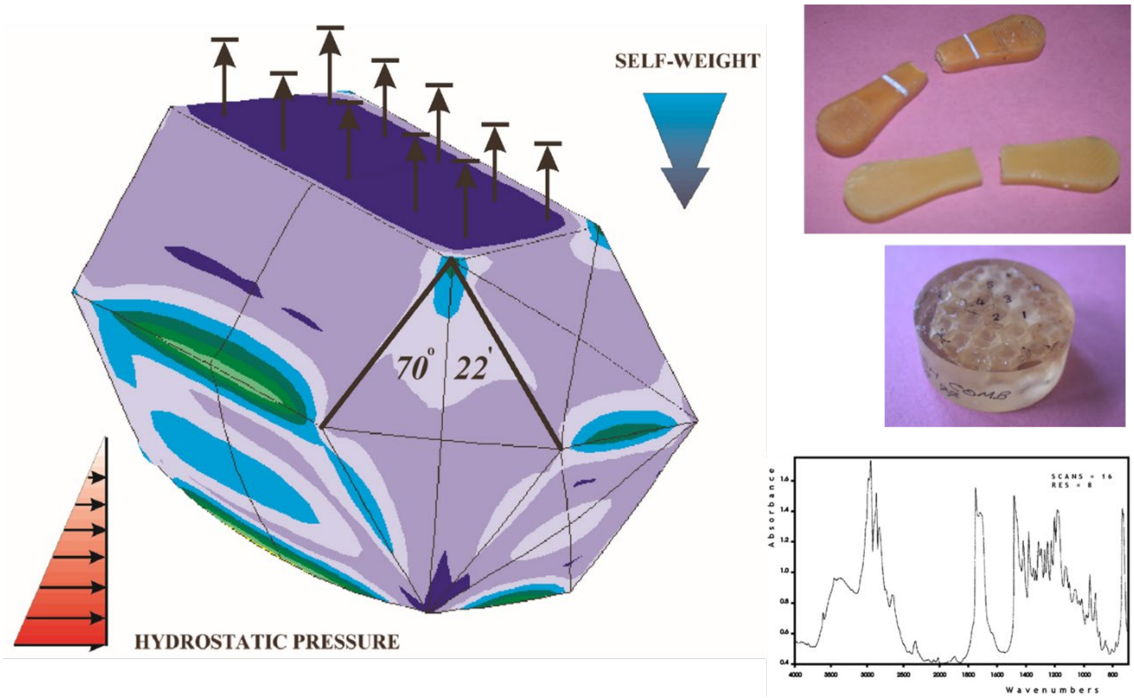


Figure 3 : Minimum stress solution for honeybee cell

It may be noted that any differences between the above angles are within the convergence tolerance set in the analysis system used.

Also shown in this figure are indications of some ancillary studies on the comb. Studies carried out on a coordinate measurement machine gave a good indication of variability in shape and thickness of the comb. As with natural structures in general, precision is not as high as what could be achieved in man-made products. Tensile tests on wax specimens provided a representative value of modulus for modelling. Interestingly, subsequent re-melting and re-casting of wax tensile test specimens, led to a significant change in the load-deflection characteristics. Chemical analysis of the material before and after re-melting indicate a loss of chemical elements and this was concluded to be the cause of the change.

In 1781 Glaisher[11] discussed the work of L'Huillier, who had extended the problem to consider the minimum minimorum cell and examined the proportion of the depth of the cell to its width, for a given volume and minimum wax. Similar variations to the problem have also been reported more recently [12][13]. However, it is true to say that such variations, while they may be of interest from a mathematical viewpoint, neglect the fact that in a feral nest, the cells are also used to rear brood as well as to store honey and pollen. The cell width and depth is therefore related to the size of the bees themselves and whether the cell is to be used to rear workers or drones. The *problem of the bees* therefore starts from the premise that the cell width and depth are fixed and that the bottom of the cell is to be closed by three equal rhombs.

I will close this section with a cartoon, which I have modified slightly …

Figure 4 : A Bee Cartoon

The original cartoon involved a *Fred* rather than a *Freda*. Beekeeping readers will no doubt know that it is the unfertile females in the hive that do all the work!

3. Elimination of Stress Concentrations … *Trees and the Baud Curve.*

At the end of the last century and the beginning of this century, the curves and shapes that appear in nature held a particular fascination for mathematicians, biologists and natural historians of the time[5][14]. Treatises on logarithmic spirals and catenary curves abound! A particular curve that is extremely common in nature was reported by Baud in 1934[15] and was also discussed by Peterson[16] in relation to reducing stress concentration effects at changes of section. The curve is described as a form of fillet based upon the contour produced by an ideal frictionless fluid flowing by gravity from a circular opening in the bottom of a tank, as shown in Figure 5.

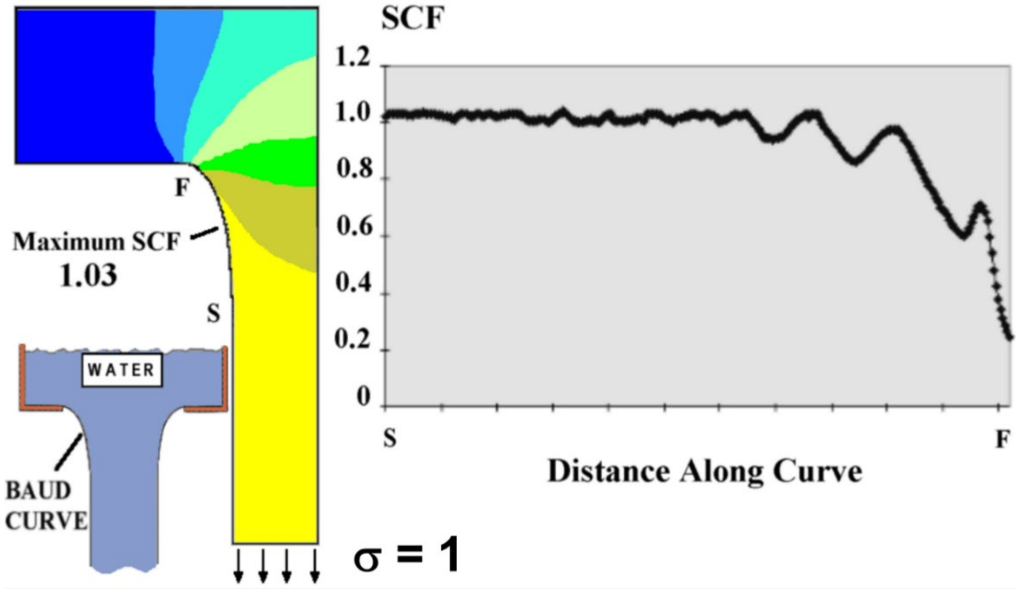


Figure 5 : Stresses for a Baud fillet

To illustrate the weight efficiency of the Baud curve, a stepped shaft subjected to an axial tension was examined. The contours of maximum principal stress are shown in Figure 6 for the case of a reference circular fillet, **with the same swept volume**.

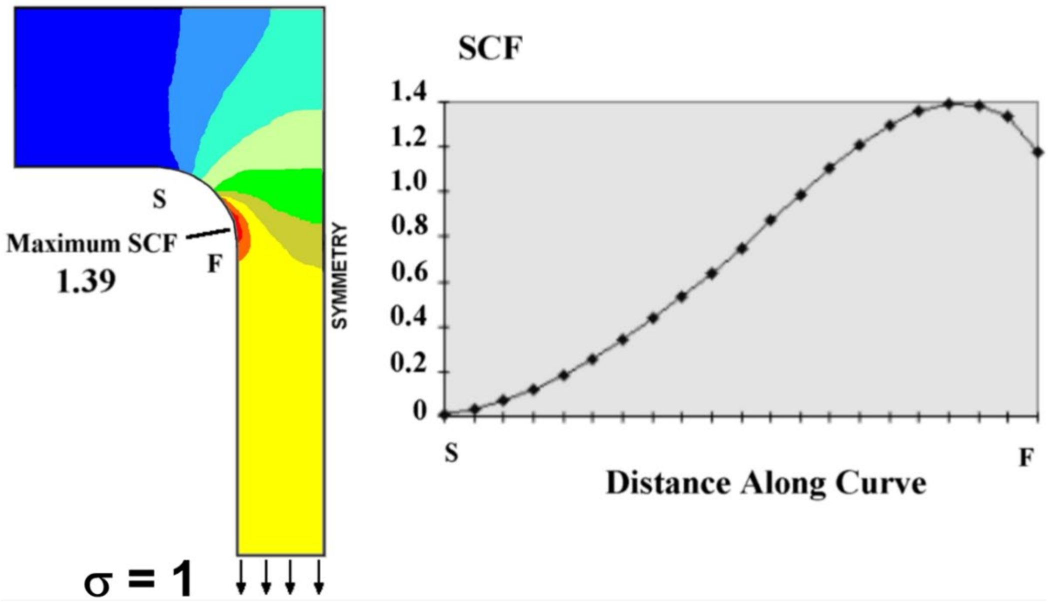
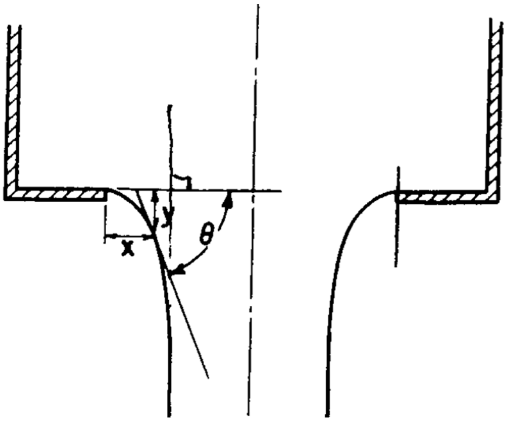


Figure 6 : Stresses for a reference circular fillet

It may be observed from these results, that use of the Baud curve has produced a 26% reduction in the magnitude of the stress concentration factor.

Circular fillets in nature are a rare occurrence and the reasons are perhaps apparent. Baud curves in Nature on the other hand are very common, as illustrated in the trunk and branches of the tree shown in Figure 7.



$$x = 2\frac{d}{\pi}\sin^2\frac{\theta}{2}$$

$$y = \frac{d}{\pi}\left[\log\tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right) - \sin\theta\right]$$

Figure 7: Baud curves in trees (Isle of Bute)

Topping [17] used the growth reforming technique to examine such a profile in a tree trunk and Mattheck [18] has studied the shape extensively in relation to trees and bones.

The routine production of Baud curves in the machining of engineering components is clearly not as simple as a circular fillet due to the more complex profile. In addition, the Baud curve is theoretically asymptotic to infinity, which could cause difficulties if bearings or gears must locate against shoulders. This could largely be overcome if geometric modellers catered for Baud curves and it would be possible to truncate the asymptotic part of the Baud curve a short distance from the shoulder, with an acceptable increase in stress. Use for castings and forgings would be less problematic, as would use of CNC machining.

Interestingly, Bartsch et al [37] recently identified tree structures as a promising approach to support structures for Additive Manufacture using Selective Laser Melting. It was concluded that they are able to withstand mechanical as well as thermal load … driving this industry sector closer to a right-first-time outcome. These tree-like structures, shown in Figure 8, came about as a result of thermal topology optimization studies.

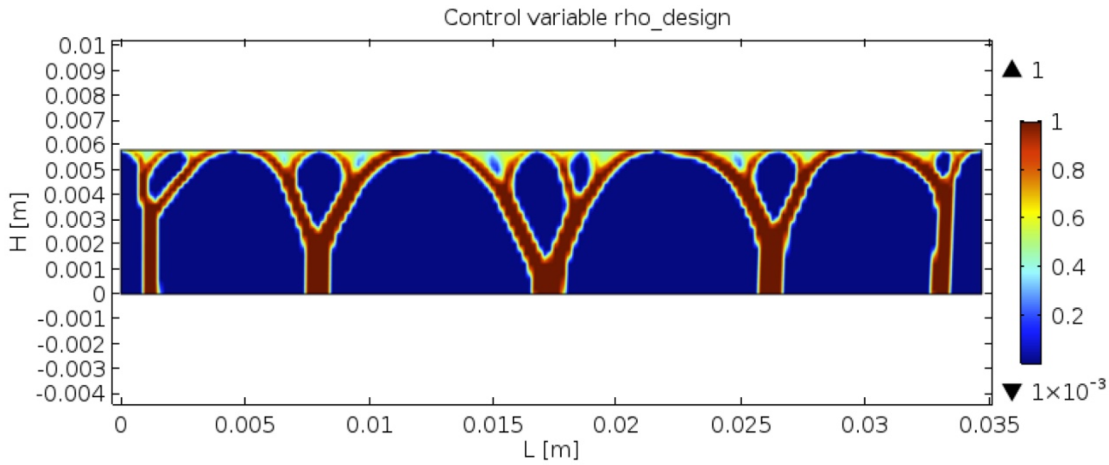


Figure 8 : Schematic representation of web showing forces (Reproduced from Bartsch et al [37])

4. Some Observations … *On Spider's Silk and Webs.*

The spider's web illustrates several different examples of minimisation. D'Arcy Thompson in his fascinating discourse on soap bubbles and raindrops [6] also notes that the beads of adhesive or dew on such webs are examples of minimum surfaces.

In the last century, mathematicians gave considerable attention to such surfaces and D'Arcy states as a fundamental law of capillarity that a liquid film in equilibrium assumes a form that gives it a minimal area under the subjected conditions. On this basis he describes how, under surface tension effects, the liquid coating on the web first of all changes to an unduloid and then finally to the string of spherical beads that are so obvious on dew covered orb webs. The orb web is essentially a planar tension structure which consists of structural elements (guy, frame, chords and radii), composed of thread drawn from the spider's ampullate glands.

Each structural member is also comprised of differing numbers of strands, with the guys being thickest. The spiral thread is coated with adhesive material from the spider's aggregate glands. The actual construction of the web is quite a fascinating process and it has been found that it is built without either visual feedback or reference to gravity (but is affected by the drug LSD) [19][20]. It is interesting to observe that our knowledge of the spider's complex behaviour and the sensory and motor apparatus which underlies the web building behaviour seems to exceed our knowledge of the somewhat similar activities of the honey bee in building the comb structure.

This is perhaps a reflection of the fact that the bee activity occurs in the darkness of the hive, within a cluster of active bees and is therefore more difficult to observe.

It is not untypical for a web to be constructed in under half an hour, use 20m of 1-3µm diameter thread and for an entire web to weigh between 0.1 and 0.5mg. The spider itself may weigh in excess of 500mg, although 100-150mg is typical for Araneus.

The structure of the orb web itself has also been examined in terms of its weight efficiency. Wainwright et al [8] present and discuss the force system in an abstract web, shown in Figure

9, when the guys are subjected to a tension of 2 units. The authors point out that this produces a remarkably uniformly stressed structure, given that the number of threads in each radius, frame and guy element is 2, 10 and 20 respectively. There is some natural variability in these figures [21], but the observation would appear to be generally valid nonetheless.

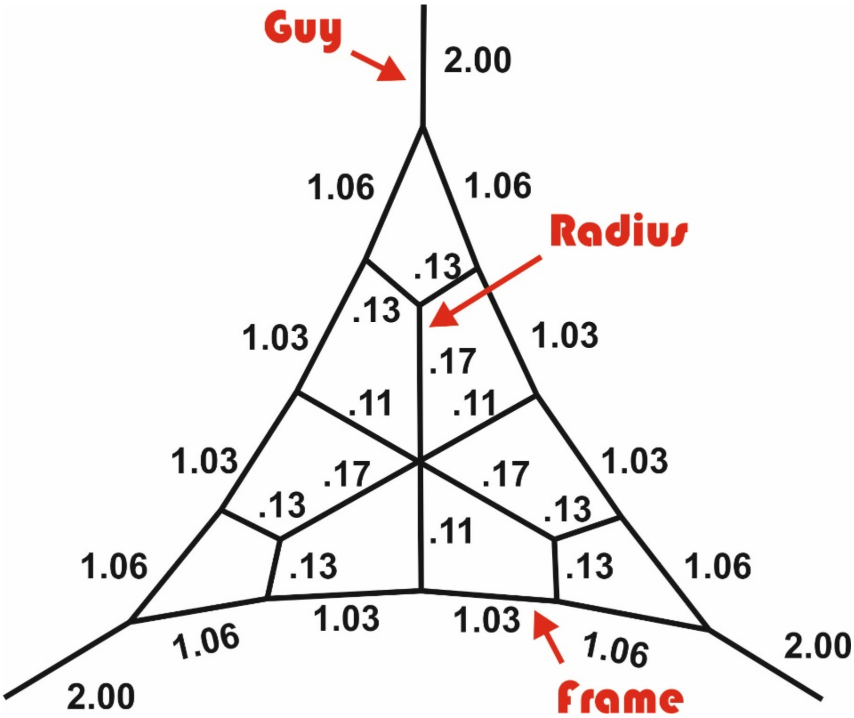


Figure 9 : Schematic representation of web showing forces (Reproduced from Wainwright et al [8])

Shown in Figure 10 is a finite element model of a symmetrical section of the web. The structure was assumed to be composed of pin-jointed tension bar elements.

A linear elastic, small displacement, optimisation problem was set up in which the cross sectional area of each element was a variable. The goal was set as minimum mass, with the constraint that the tensile stress in each element should be the same.

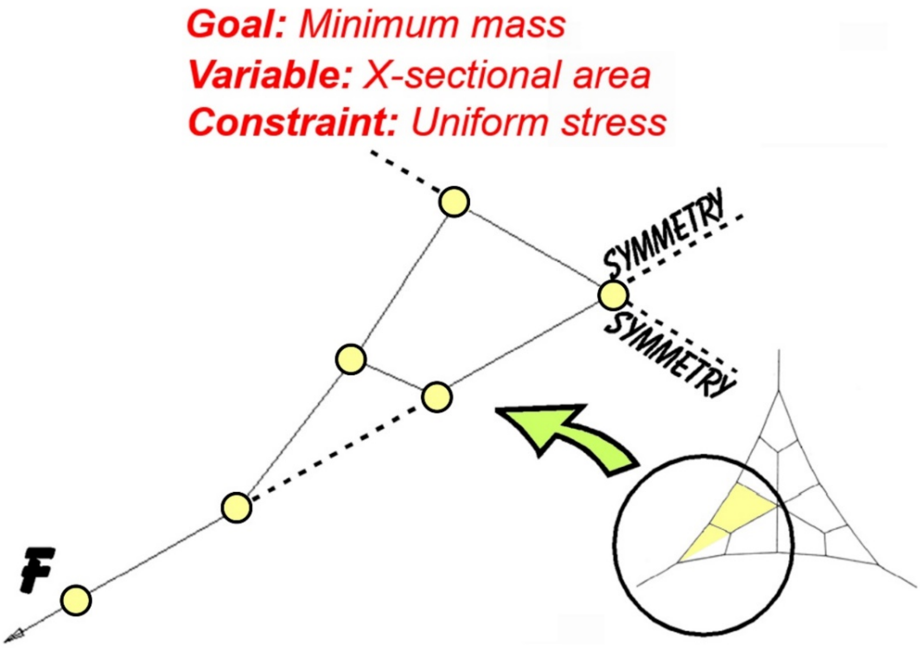


Figure 10 : Finite element model of web section

The results from the study are shown in Figure 11 and it may be observed that there is quite good agreement with Wainwright et al ... given the accuracy of the input web data. Other interpretations of available data [9][21] show approximately a factor of 2 variation in stress throughout such webs with the thinner radial elements showing the greatest variation. The question arises whether this is due to natural and/or experimental variation, or whether it is due to evolved differences in factors of safety for the various web elements.

WEB SECTION	Nominal Area from FE study	Nominal Area from Wainright et al
Guy	20	20
Frame	10.8	10
Radius	1	2

Bundles of silk threads vary in diameter to give a uniform stress!

Figure 11 : Comparison of web section cross-sectional areas

The silk used in the web construction is no doubt a remarkable material, given the constraints on the spider's method of production. Its method and rate of production is also impressive and has been widely studied [19][20]. Denny[21] reported that the framework threads of Araneus Seracatus have on average, a true breaking strength of around 1GPa, a tangent modulus of 4GPa and a corresponding breaking strain of approximately 0.25. The sticky spiral threads, that have to deal with the struggling prey on the other hand, have the same breaking strength, a tangent modulus of 0.6GPa, and can withstand strains up to 2. The framework silk was found to be strain rate dependent, whereas the viscid spiral silk was found to have properties that were insensitive to strain rate. These significantly different properties are produced from materials that are chemically very similar.

Like many natural materials and structures, there is apparent evidence of purpose and conscious design and the sticky spider silk exhibits further amazing characteristics, as shown in Figure 12. Inside the adhesive globules, the silk is coiled, which would result in a J-shaped load-deflection characteristic. This in turn will arguably influence the effectiveness of the web at capturing and holding struggling prey, whilst avoiding damage. Also shown in the figure are a typical set of spinnerets, which can produce up to 7 different types of silk in some species.

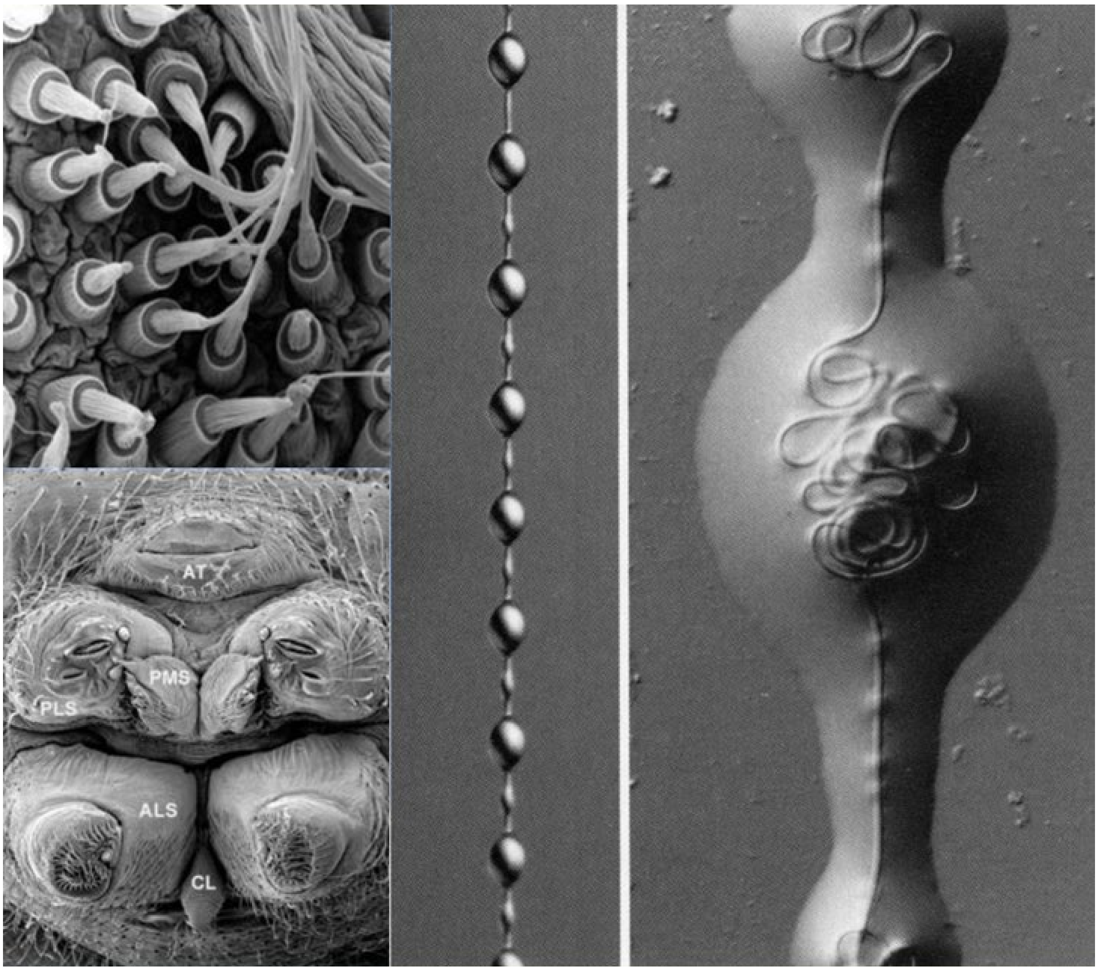


Figure 12 : Detailed images of spider silk and spinnerets

When the engineering tensile strength of the framework silk is evaluated per unit density, it is found that the silk (σu =0.8GPa & SG=1.26) is approximately 12 times stronger than typical carbon steel (BS4360 Gd43 : σu =0.43GPa & SG=7.8) and 4 times stronger than aerospace standard high strength aluminium alloy (BS2L93 Gd 2014A : σu =0.41GPa & SG=2.7).

Also noteworthy is the fact that many such spiders tear down and rebuild their web daily and to reduce the metabolic cost, they eat the old web!

Whether this is an optimum structure or not depends on how the problem is posed. It is certainly approaching a least volume structure as defined by Maxwell's Lemma [8], in that all members are in pure tension and are equally stressed near their breaking stress. However, there are spiders that manage to catch prey and survive as a species with a greater economy of silk ... in fact there are species of spider that do not rely on a sticky web to capture prey (eg the Bolas spider utilizes a length a silk with a sticky blob at the end)! As mentioned earlier, identifying the optimum solution and indeed the optimization problem itself, is not always easy in nature.

5. A Remarkable Lightweight Beam … *The Rachis of a Bird Feather.*

The rachis of a bird feather effectively behaves like a cantilever beam as part of a wing. It is in effect supported along its length by adjacent feathers, but for the purpose of explaining structural function and form, its characteristics were considered as a cantilever (built-in at one end). The wing and flight of birds also have numerous other interesting characteristics. The rachis material is Keratin, which is also found in hair and nails. From a functional viewpoint, the rachis structure must be lightweight, whilst facilitating growth and flight. Videler [25] provides an excellent coverage of wing morphology, including flight feathers, aerodynamics and energetics.

While this exemplar doesn't have the historical significance of the honeycomb of bees, it has been of interest for some time … the Oxford University Museum of Natural History, founded in 1860, has a display of the remarkable structure of the rachis.

For the present study, students regularly acquired the wings of dead birds for dissection and study in the materials laboratory. In Figure 13 we see a historical presentation of the rachis cross-section at various positions along its length. Alongside this are SEM images, and different adaptive-p finite element models (from different birds), simplified as necessary in a staged programme aimed at understanding better the function of different features. The mechanical properties of the internal cellular structure were obtained from small specimens tested in an attachment to a scanning electron microscope.

From an engineering structural viewpoint the following points are worthy of note:

(a) The cantilever cross-section is tapered, which reflects the reduction of bending load from tip to root.
(b) The sections however vary enormously in shape and form along the length, whilst retaining a reasonably symmetrical section about the vertical. This would perhaps indicate that the feathers are not individually subjected to a high degree of torsion.
(c) The feather starts with an oval hollow cross section for the section within the bird that is connected to the muscles. The thickness of the oval is not uniform.
(d) For the majority of the length, the cross-section is composed of an outer Keratin shell of complex profile, with an internal spheroidal (not tubular) cellular Keratin structure … thus providing stiffness in a weight-efficient manner. There is no dissimilar material joining involved between the shell and cellular structure, as the material is the same (albeit with different form).
(e) The top surface of the shell develops what appear to be longitudinal stiffeners on

the inner surface. This would be the surface subjected to the highest compression (during the down flap of the wing) and would help resist buckling.

(f) On the bottom (ventral) surface of the shell there is a thin notch / "hinge-like" feature, which will result in a bi-directional variation in stiffness … which is also thought to reduce the effort required on the up flap of the wing. This same feature, whilst undergoing buckling and large displacement, is thought to provide an increase in section stiffness on the up flap.

There are no doubt further specific aerodynamic explanations that could arguably be related to function and form?

An extremely impressive beam no doubt!

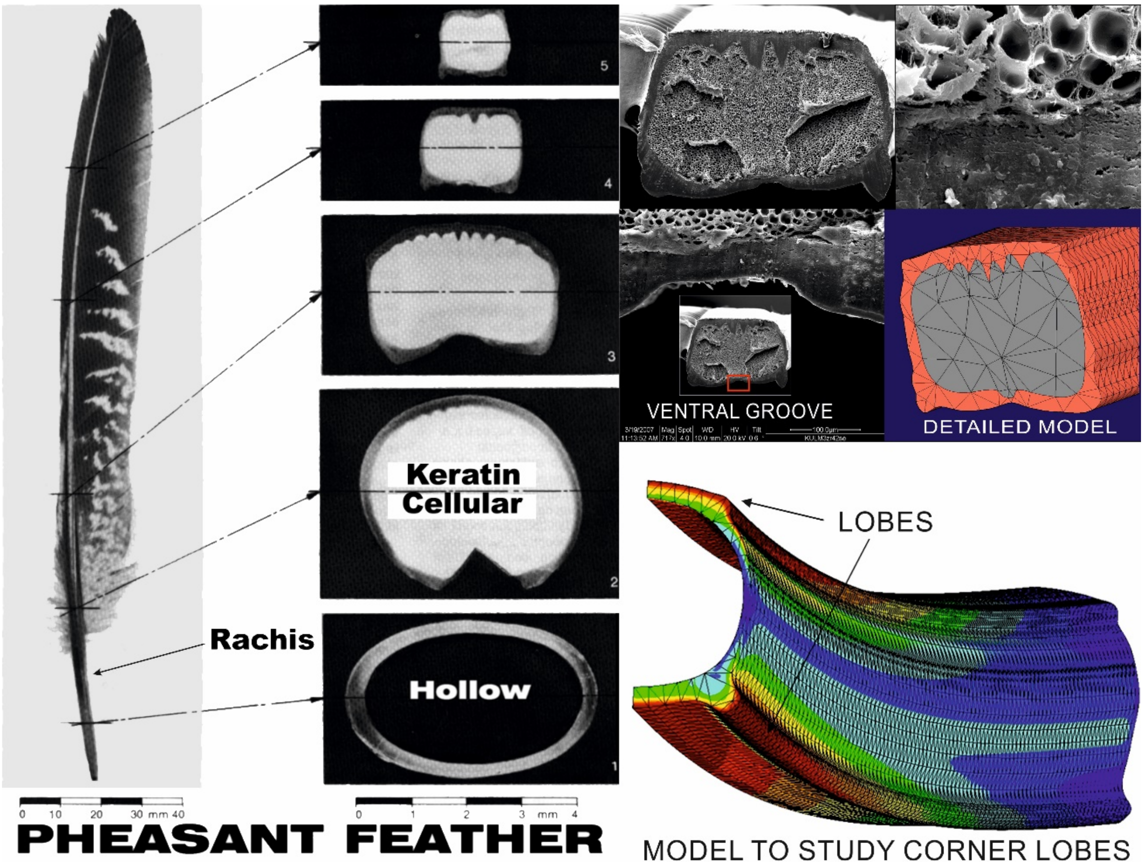


Figure 13 : Bird Feather Rachis Specimens & Models

Wing structures in Nature are diverse indeed, as the SEM image of a Damsel Fly wing shown in Figure 14 shows. This looks similar to a steel fabricated structure that would not be out-of-place in shipyards throughout the world. The Damsel Fly no doubt blissfully unaware as to the engineering purpose of the gusset stiffeners!

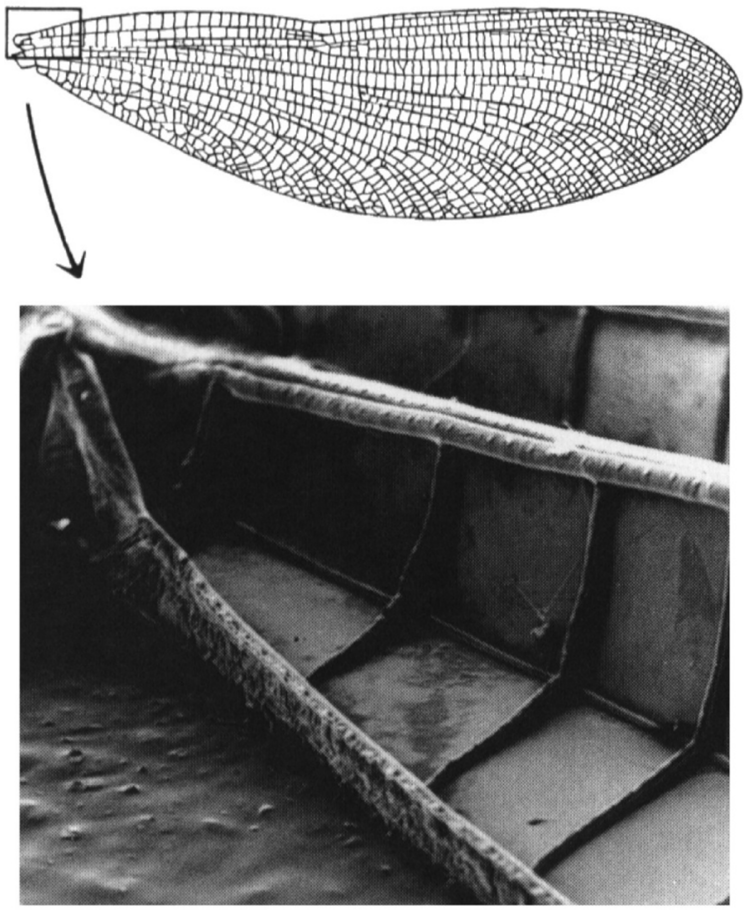


Figure 14 : Pleating on the Wing of a Damsel Fly

6. An Extraordinary Laminated Composite … *A Human Heart Valve Leaflet.*

This example stems from collaborative work with David Wheatley and his team at the Glasgow Royal Infirmary, where a polyurethane replacement heart valve was being developed. Figure 15 shows a nonlinear buckling model of a tri-leaflet polyurethane heart valve leaflet. The polyurethane was modelled in Abaqus as a hyperelastic homogeneous material, which is a simplification of its viscoelastic time and strain-rate dependent behavior. In addition, a coupled non-Newtonian fluid-structure interaction was not included. Despite these simplifications the 2 stage buckling of the doubly-curved leaflet was predicted and a reasonable indication of the high strain-rates present in the rapid opening and closing of the valve was identified.

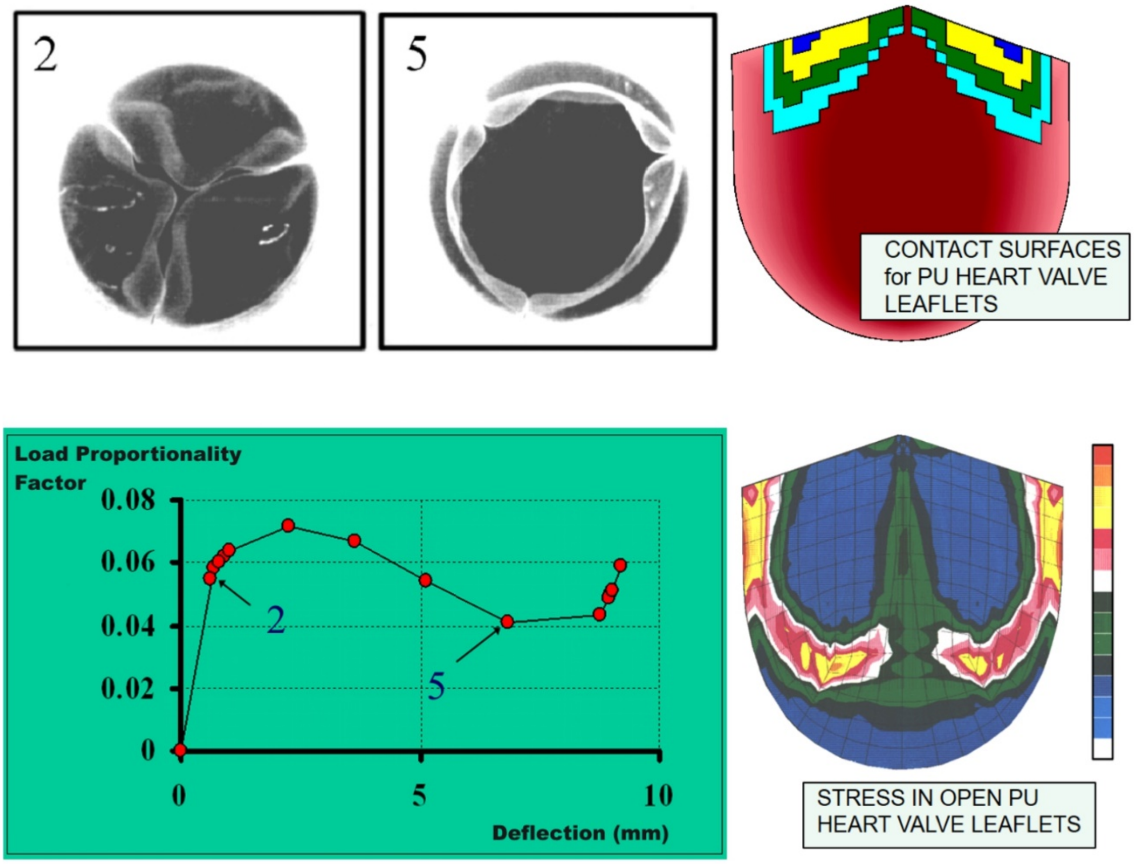


Figure 15 : A Buckling FE Model of a Polyurethane Heart Valve Leaflet [27]

When replacing a human organ with a man-made component, it is natural to look at the material and function of what is being replaced.

Figure 16 shown below shows a highly complex laminated composite construction, that breaks many of the rules often applied with traditional engineering composite construction.

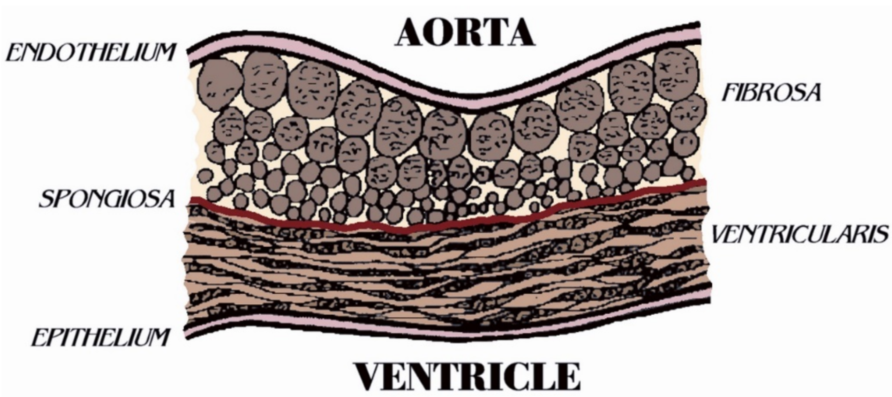


Figure 16 : Composite Construction of a Human Heart Valve Leaflet [26]

Observations on complexity:

(a) There are 2 main structural plys in an entirely unsymmetrical layup (the Fibrosa and Ventricularis).

(b) The Fibrosa ply is reinforced by bundles of unidirectional Collagen fibres, with the bundle diameters changing significantly from the Endothelium to the Spongiosa i.e. effectively reinforcing fibres of different diameters in the same layup.

(c) The Ventricularis on the other hand is made up of a 0/90 layup of Collagen and Elastin fibres i.e. 2 different types on reinforcing fibres in the same layup.

(d) The fibrils in such reinforcing fibres are generally crimped (linear or helical), which is thought to contribute to the J-shaped load-deflection curve characteristic in many biological materials.

(e) The Spongiosa layer between the 2 structural plys is reported as being more like a gel and is thought to provide interply lubrication as the leaflets undergo large displacement during opening and closing of the valve. Another theory is that it provides a damped response during the sudden opening and closing. Interestingly, when tested in a pulse duplicator, the man-made polyurthane leaflets exhibit a typical damped vibration response. Leaflets harvested from dead animals on the other hand show little or no vibration.

Trying to explain the purpose of such complexity and to define an optimization scenario that led to such complexity, would be challenging indeed. In fact, as an engineer, it can be all too easy to develop a feeling of inadequacy!

Current optimization software systems would no doubt struggle to arrive at such a conclusion for such a complex laminated composite structure.

Composite material construction appears widely in biological material and structures. In addition to the complexity and functionality, it is not unusual for such material and structures to adapt throughout their life, to grow whilst still functioning, to feel pain, to self-repair etc. Human skin is another remarkable material. Clearly in this area alone, there is much to inspire engineers.

7. Interesting Shell Structures … *Constant Strength & Logarithmic Spiral.*

Constant strength shell structures would appear to be an example of something that was simple enough to be understood, designed and manufactured by engineers, as shown in Figure 17.

Such storage tanks are certainly not popular and apart from the cost and complexity of manufacture, the shape is only an optimum when the tanks are full. As Timoshenko demonstrates [28], this is also the shape of a water droplet lying on a frictionless surface. The idea for this shape of storage tank is attributed to the Chicago Bridge & Iron Works [29]. The behavior of underwater enclosures of optimum design was reported by Royles et al [30] and the similarity to a sea urchin is discussed. It is concluded however that because the urchin is a free flooding animal, its shape did not arise in response to a hydrostatic pressure. The lesson being, that in this case the inspiration came from the shape of a water droplet and not a sea urchin … in the world of Bioinspiration, appearances can be deceptive! That is not to say that the shell of the sea urchin does not hold other lessons for engineers. Our early historical attempts to fly like a bird are often cited as an example of mimicry, without understanding.

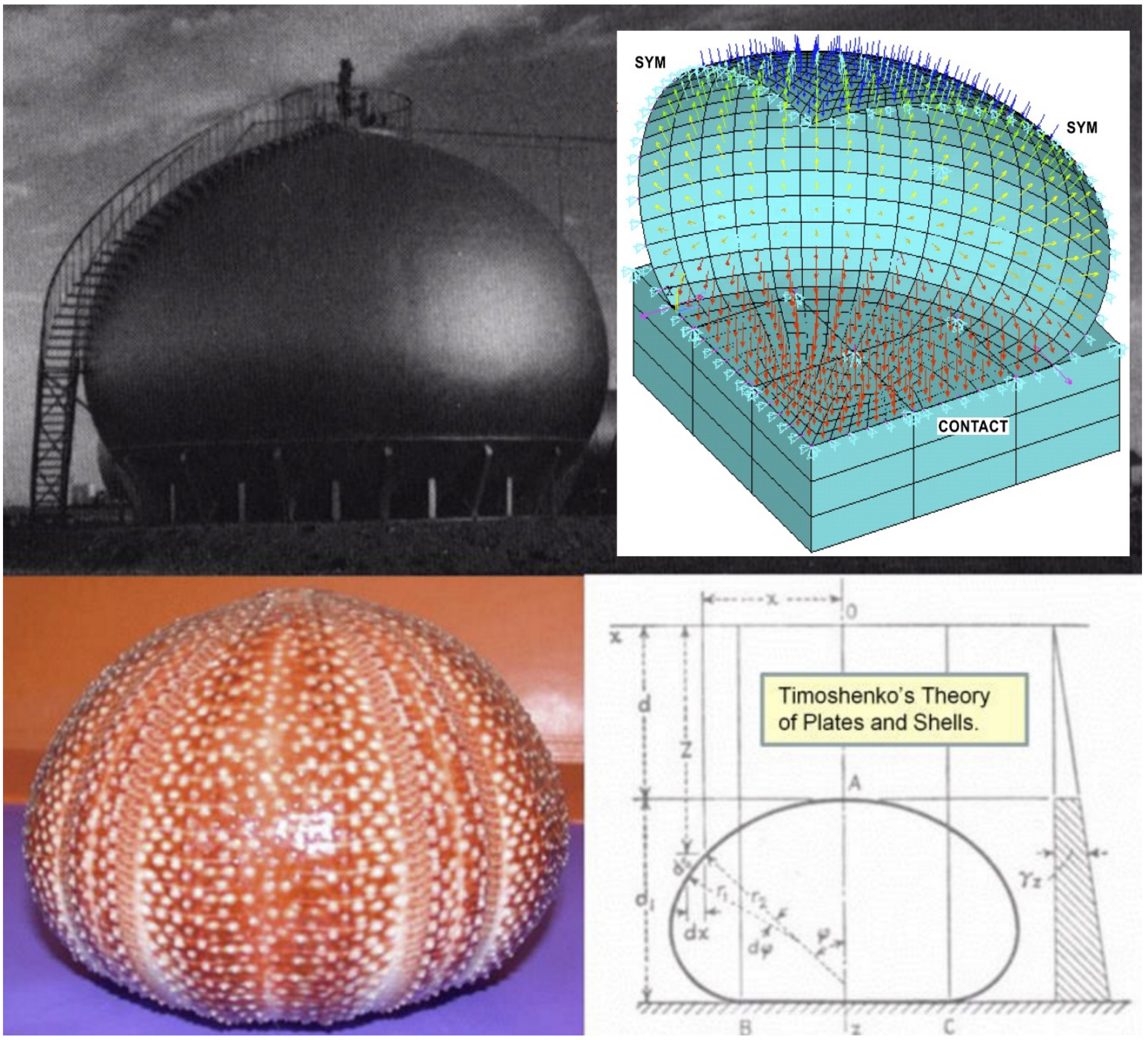


Figure 17 : Constant Strength Shells

Shells have also inspired work on improving the toughness of ceramics, using an understanding the relative toughness of sea shells, made from what on the face of it is an unpromising basic material [31]. The Nautilus shell, which is a Logarithmic Spiral form (another of the Curves of Life [14]) has inspired work on the potential of structures to grow with the needs of inhabitants. Mattheck [23] has also argued that the logarithmic spiral has inherent strength properties. Shown in Figure 18 are an example of a logarithmic spiral finite element model, created using a Visual Basic programme, capable of producing a wide range of logarithmic spiral shells, using the spiral parameters provided. Also shown are sections of a Nautilus shell, used for geometry and thickness measurements.

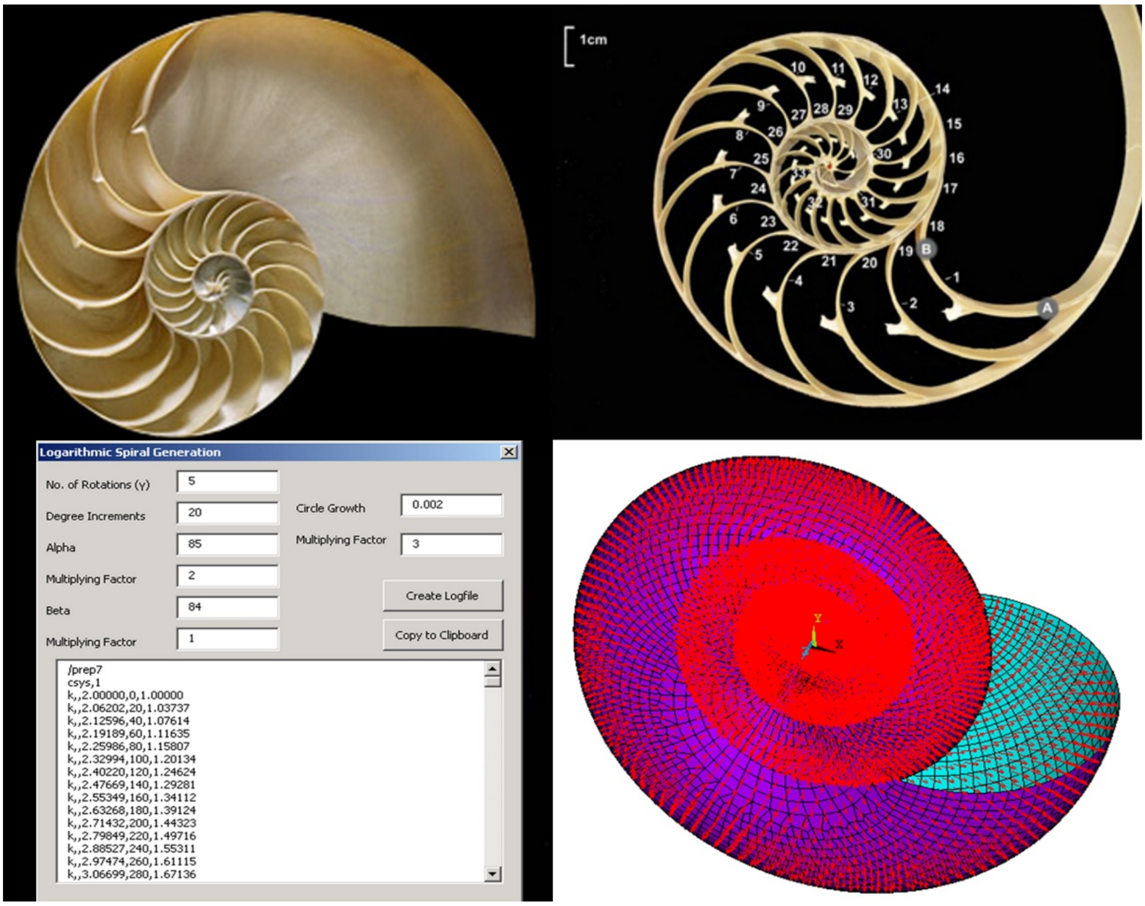


Figure 18 : Nautilus Shells

8. Some Thoughts on the Original Question

Clearly there would appear to be a purpose to all of the examplars presented herein. I have however used the term **fitness for purpose** as opposed to **design**. But the distinction between the 2 really cant be avoided. Design implies conscious thought and much of what you see in Nature is so fit for purpose that this has been a major and continuous source of belief over the centuries that there must indeed have been a creator. This debate is still raging today and there are many relevant texts. Those interested in both sides of the arguments could start with references [33][34].

In 1826, William Paley [32] published a seminal contribution that no doubt articulated the thoughts of people down the millennia. His famous opening paragraph is reproduced below and this led in turn to equally famous texts by Richard Dawkins [34] presenting the contrary view.

In crossing a heath, suppose I pitched my foot against a stone, and were asked how the stone came to be there: I might possibly answer, that for any thing I know to the contrary, it had lain there for ever: nor would it perhaps be very easy to show the absurdity of this answer. But suppose I had found a watch upon the ground, and it should be inquired how the watch happened to be in that place; I should hardly think of the answer which I had before given, that for any thing I knew, the watch might have always been there. Yet why should not this answer serve for the watch, as well as for the stone? Why is it not as admissible in the second case as in the first? For this reason, and for no other, viz., that *when we come to inspect the watch, we perceive (what we could not discover in the stone) that its several parts are framed and put together for a purpose ... This mechanism being observed ... the inference, we think, is inevitable, that the watch must have had a maker; that there must have existed, at some time, and at some place or another, an artificer or artificers, who formed it for the purpose which we find it actually to answer; who comprehended its construction, and designed its use.*

As has been pointed out by Dawkins, Paley was apparently not the first to make this argument using a timepiece metaphor … Cicero (106BC-4sBC) had done so along with quite a few others. Paley also made the point that Nature's / the Deity's creations are far more complex and impressive than a watch!

The observant reader will already appreciate that the above examples represent but a small fraction of the fascinating array of *highly fit for purpose* structures readily available in nature. While some optimization problems may be too difficult to formulate, others would no doubt make a worthwhile and interesting addition to traditional shape optimization benchmarks. In addition, it is also probable that most of the structural finite elements, from one dimensional through to three dimensional, could be accommodated.

It is the widespread occurrence of free-form curves in such natural structures that presents the challenge to shape optimization systems developed for engineering purposes. However, in reality, representing the true behaviour of many of these structures and their constituent materials would also present severe challenges, in that complexity - non-linear and time-dependent behaviour is common. This need not present an insurmountable problem in the present context and it is likely that many interesting and varied problems could also be posed through the simplifications of assuming small displacement behaviour and a linear elastic, homogeneous and non time-dependent material. The general optimization problems associated with biological systems, which could often theoretically be classed as constrained, stochastic, non-linear, multivariable and dynamic, are also invariably simplified and posed as special cases. It is likely however that true fitness for purpose (and also a basic understanding of function), is inherently linked to the complexity of the material and structural behavior.

In conclusion, the list below provides a summary of thoughts on the original question posed …. *Are Biological Structures Optimum?*

(1) The fact that a biological entity survives today is evidence of being, at the very least, of adequate fitness for purpose. It would certainly appear from the small sample herein, that the word *adequate* would not generally do them justice!
(2) Depending on your viewpoint, because of God and/or the pressures to be fitter for the purpose of surviving (possibly both), biological structures and materials provide a wealth of examples of *fitness for purpose* and *weight efficiency*.
(3) The interpretation of *design* in Nature shall probably range from being simply adequate to being a local optimum, with many tending towards the latter category, were we able to understand the optimization scenario.
(4) Whether optimum or not, many engineers and scientists shall no doubt be content to draw inspiration.

The following list provides a convenient, unstructured, list of thoughts and observations.

(a) Transferring lessons into the engineering domain is often a challenge (e.g. the reproduction of spider silk) and it would be unreasonable to expect Nature to supply direct lessons in all we do.
(b) Hopefully the beauty and complexity of Nature's *designs* will not lead to feelings of inadequacy.
(c) The assumed goal for all organisms is commonly to *survive* and *reproduce*.
(d) Is an atomistic approach appropriate i.e. for the whole to be an optimum, does the parts have to be?
(e) Natural selection leads to organisms and structures better fitted to the environment.
(f) Because there is a cost associated with material, size and movement, many biological structures provide good examples of weight efficiency.
(g) Organisms evolve in local environments (no optimum spider for example).
(h) It is not always obvious that an optimum (or even good) solution has evolved, as the purpose and constraints are sometimes difficult to understand.
(i) Many *designs* have evolved over large timescales and yet many organisms have not apparently changed for millions of years. For most, environments and survival pressures change and hence the pressure to adapt generally continues.
(j) Unlike humans, revolutionary re-design is not possible and hence any feature / organism must be considered as a local optimum only at best?
(k) Adaptive change in short timescales is common (e.g. during the life of an organism) but no doubt many engineers (even atheists) will struggle to accept that random mutations is the sole mechanism how beneficial adaptive changes get into the genes. The complexity and apparent fitness for purpose can be so impressive.
(l) Natural structures fail and the concept of Factors of Safety is an interesting one to ponder [9].
(m) Biological solutions are restricted to materials that can be synthesized around ambient temperature (also generally readily degraded however) … unless we consider humans as part of Nature's solutions! Nature doesn't have the full range of materials and processes at its disposal that humans have.
(n) There is an *Infinite Varietie* of materials and structures in the Natural World … generally composite and hierarchical, often with no clear distinction where material and structure starts and finishes.
(o) Biological materials, structures and systems are usually complex and behavior is often nonlinear (material, strains & displacements) and time-dependent.
(p) High strains are common and stresses may be low … but strain energy can be high.
(q) Materials are generally quite tough and fracture mechanics lessons abound.
(r) A J-shaped stress-strain curve is common and many authors have reflected on the benefit of this.
(s) Pre-stressing is common … also as part of controlled and adaptive growth.
(t) Ultimate material properties are often not exceptional, but specific ones can be impressive. Solutions in Nature invariably exhibits qualities that many humans would regard as *ingenious design*.
(u) Biological materials and structures often show a high degree of tailoring and fitness for purpose.
(v) Grading of properties between dissimilar materials is common (no abrupt changes in modulus).
(w) Energy efficiency is often argued as a primary driver, due to *cost* considerations.
(x) Biological materials and structures have to accommodate growth from birth and

all the while continue to function … adaptation and repair is also common.

(y) Biological organisms can often feel *pain* and can sometimes change properties by adjusting chemistry in relatively short timescales eg the modulus of the female cervix before and after birth [36].

(z) We should of course remind ourselves that humans have created inventions and ventured into operating environments that the rest of Nature has not.

JE Gordon said that *nothing attracts less curiosity than total success*. Biological materials, structures and systems are generally perfect examples of this and most engineers probably give little thought to such matters. It is a sobering thought however that it is the mechanical failure of these, in the body, that will often lead to incapacity and death of us all. This general topic is likely to force itself on you sooner or later! ☺

Most of the work presented herein is unpublished and was generated as part of my supervision of university student projects (mainly undergraduate, although the work on heart valves was with a PhD student - Steven Barsanti) [27]. Typically this would annually involve 6off final year undergraduate individual projects and 3off group projects, I would therefore like to close by acknowledging the assistance given by all these students listed below, whom I trust were as inspired as I was.

In closure, I would also like to dedicate this article to the Scottish scholar naturalist D'Arcy Wentworth Thompson 1860-1946 … ***Hic erat vir!***

Undergraduate students who contributed to my interest in Bioinspiration over the years:
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References

[1] WOOD, J - Shape Optimisation in Biological Structures, Invited Lecture, Proceedings of the NAFEMS World Congress on Effective Engineering Analysis, Newport, Rhode Island, USA, April 1999.
[2] FRENCH, M. - Invention and Evolution, Design in Nature and Engineering; Cambridge University Press, 2nd Ed., 1994.
[3] DARWIN, C. - The Origin of Species by Means of Natural Selection or the Preservation of Favoured Races in The Struggle for Life; Oldham Press, 6th Ed., 1872.
[4] DAWKINS, R. - The Blind Watchmaker; Longman, 1986.
[5] CAJORI, F. – Sir Isaac Newton's Mathematical Principles of Natural Philosophy and His System of the World, v2, University of California Press, 1934.
[6] THOMPSON, D.W. - On Growth and Form; Cambridge University Press, 2nd Ed., 1942.
[7] ALEXANDER, R.Mc. - Optima for Animals; Edward Arnold, 1982.
[8] WAINWRIGHT, S.A. et al - Mechanical Design in Organisms; Edward Arnold, 1st Ed., 1976.
[9] ALEXANDER, R.Mc. - Factors of Safety in the Structure of Animals; Sci. Prog.,

Oxf., 67, pp109-130, 1981.
[10] MacLAURIN, C. - Of the Bases of the Cells wherein the Bees Deposit their Honey; Phil Trans, p42, pp561-571, 1743.
[11] GLAISHER, J.W.L. - On the Form of the Cells of Bees; Phil Mag, v4, pp103-22, 1873.
[12] TOTH, L.F. - What The Bees Know and What They Do Not Know; Bull Amer Math Soc, 70 (1064), pp467-481, 1965.
[13] HILDEBRANDT, S. et al - The Parsimonious Universe; Springer-Verlag, 1996.
[14] COOK, T.A. - The Curves of Life; Dover Publications, 1st unabridged republication, 1979.
[15] BAUD, R.V. - Beitrage zur Kenntnis der Spannungsverteilung in Prismatischen und Keilformigen Konstruktionselementen mit Querschnittsubergangen; Eidgenoss, Materialpruf, Ber 83, Zurich, 1934.
[16] PETERSON, R.E. - Stress Concentration Design Factors; Wiley, 1953.
[17] TOPPING, B.H.V. - Fully Stressed Design of Natural and Engineering Structures; Natuerliche Konstructionen, Proc. Sonderforscungsbereiches 230, Stuttgart, 2, pp311-318, 1989.
[18] MATTHECK, C. - Engineering Components Grow Like Trees; Institut fur Material-und Festkorperforschung, Report KfK 4648, Kernforschungszentrum, Karlsruhe GMBH, Karlsruhe, November 1989.
[19] CHINERY, M. - Spiders; Whittet Books, 1st Ed., 1993.
[20] WITT, P.N. et al - A Spider's Web, Problems in Regulatory Biology; Springer-Verlag,1956.
[21] DENNY, M. - The Physical Properties of Spider's Silk and Their Role in The Design of Orb Webs; J. Exp. Biol., 65, pp483-506, 1976.
[22] UMETANI, Y. et al - An Adaptive Shape Optimization Method for Structural Material Using the Growth Reforming Procedure; Proceedings of the Joint JSME-ASME Applied Mechanics Western Conference, pp359-365, 1975.
[23] MATTHECK, C. et al - A New Method of Structural Shape Optimization based on Biological Growth; Int J Fatigue, May 1990.
[24] MATTHECK, C. - Design in Nature, Learning from Trees; Springer-Verlag, 1998.
[25] VIDELER, J.J. - Avian Flight; Oxford University Press, 2006.
[26] VESELY, I. & NOSEWORTHY, R. - Micromechanics of the Fibrosa and the Ventricularis in Aortic Valve Leaflets; J. Biomechanics, 25(1), pp101-113, 1992.
[27] BARSANTI, S. - Observations on the Mechanical Behaviour of Polyurethane Heart Valves; PhD Thesis, University of Paisley, 1998.
[28] TIMOSHENKO, S.P. & WOINOWSKY-KRIEGER, S. - Theory of Plates and Shells; 2nd Ed., McGraw-Hill, pp443-445, 1959.
[29] DAY, C.L. - New Spheroidal Design for Large Oil Tanks; Engineering News-Record, pp416-419, September 12, 1929.
[30] ROYLES et al - Behaviour of Underwater Enclosures of Optimum Design, pp12-20, Strain, January 1980.
[31] CHENG, L. et al - Structure Design, Fabrication, Properties of Laminated Ceramics: A Review; Int. J. of Lightweight Materials and Manufacture, v1(3), pp126-141, September 2018.
[32] PALEY, W. - Natural Theology or Evidences of the Existence and Attributes of the Deity, Collected From the Appearances of Nature; 1826.
[33] HARUN YAHYA - Signs of God: Design in Nature; Global Publishing, 2003.
[34] DAWKINS, R. – The Blind Watchmaker, Penguin Books, 2006.

[35] GORDON, J.E. – The New Science of Strong Materials, Penguin Books, 1968.
[36] KOKENYESI, R. & WOESSNER, JR. – Relationship between Dilation of the Rat Uterine Cervix and a Small Dermatan Sulfate Proteoglycan; Biology of Reproduction 42, 87-97, 1990.
[37] BARTSCH, K. et al – A Novel Approach to Support Structures Optimized for Heat Dissipation in SLM by Combining Process Simulation with Topology Optimization; Proc. NAFEMS NWC, Quebec, June 2019.